



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Hypatia Contest

(Grade 11)

Wednesday, April 13, 2016

(in North America and South America)

Thursday, April 14, 2016

(outside of North America and South America)



UNIVERSITY OF
WATERLOO

Time: 75 minutes

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Do not open this booklet until instructed to do so.

Number of questions: 4

Each question is worth 10 marks

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by



- worth 2 or 3 marks each
- full marks given for a correct answer which is placed in the box
- **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by



- worth the remainder of the 10 marks for the question
- **must be written in the appropriate location** in the answer booklet
- marks awarded for completeness, clarity, and style of presentation
- a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express calculations and answers as exact numbers such as $\pi + 1$ and $\sqrt{2}$, etc., rather than as $4.14\dots$ or $1.41\dots$, except where otherwise indicated.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:

- Please read the instructions on the front cover of this booklet.
- Write all answers in the answer booklet provided.
- For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
- For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
- Diagrams are *not* drawn to scale. They are intended as aids only.
- While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x -intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
- No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

- Raisins are sold by the scoop, cup, jar, basket, or tub in the following proportions:
5 scoops of raisins fill 1 jar, 3 scoops of raisins fill 1 cup, 5 baskets of raisins fill 2 tubs,
and 30 jars of raisins fill 1 tub.



(a) How many tubs of raisins fill 30 baskets?



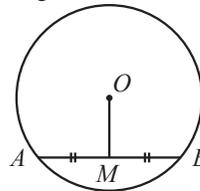
(b) How many cups of raisins fill 6 jars?



(c) Determine how many cups of raisins fill 1 basket.

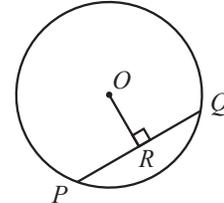
- If a line segment is drawn from the centre of a circle to the midpoint of a chord, it is perpendicular to that chord. For example, in Figure 1, OM is perpendicular to chord AB .

Figure 1

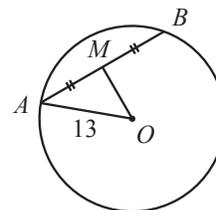


If a line segment is drawn from the centre of a circle and is perpendicular to a chord, it passes through the midpoint of that chord. For example, in Figure 2, $PR = QR$.

Figure 2



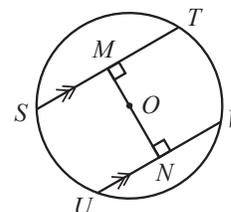
(a) In the diagram, a circle with radius 13 has a chord AB with length 10. If M is the midpoint of AB , what is the length of OM ?



(b) In a circle with radius 25, a chord is drawn so that its perpendicular distance from the centre of the circle is 7. What is the length of this chord?



(c) In the diagram, the radius of the circle is 65. Two parallel chords ST and UV are drawn so that the perpendicular distance between the chords is 72 ($MN = 72$). If MN passes through the centre of the circle O , and ST has length 112, determine the length of UV .



3. For a positive integer n , $f(n)$ is defined as the exponent of the largest power of 3 that divides n .

For example, $f(126) = 2$ since $126 = 3^2 \times 14$ so 3^2 divides 126, but 3^3 does not.



(a) What is the value of $f(405)$?



(b) What is the value of $f(1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10)$?



(c) Let N be the positive integer equal to $\frac{100!}{50!20!}$. Determine the value of $f(N)$.

(Note: If m is a positive integer, $m!$ represents the product of the integers from 1 to m , inclusive. For example, $5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$.)



(d) Given that $f(a) = 8$ and $f(b) = 7$, determine all possible values of $f(a + b)$.

4. Erin's Pizza (EP) and Lino's Pizza (LP) are located next door to each other. Each day, each of 100 customers buys one whole pizza from one of the restaurants. The price of a pizza at each restaurant is set each day and is always a multiple of 10 cents. If the two restaurants charge the same price, half of the 100 customers will go to each restaurant. For every 10 cents that one restaurant's price is higher than the other restaurant's price, it loses one customer to the other restaurant. The cost for each restaurant to make a pizza is \$5.00.

As an example, if EP charges \$8.00 per pizza and LP charges \$9.00 per pizza, the number of customers and the resulting profit for each restaurant is shown in the table below.

Restaurant	Price per pizza	Number of customers	Profit
EP	\$8.00	$50 + 10 = 60$	$60 \times (\$8.00 - \$5.00) = \$180$
LP	\$9.00	$50 - 10 = 40$	$40 \times (\$9.00 - \$5.00) = \$160$



(a) On Monday, EP charges \$7.70 for a pizza and LP charges \$9.30.

- (i) How many customers does LP have?
- (ii) What is LP's total profit?



(b) EP sets its price first and then LP sets its price. On Tuesday, EP charges \$7.20 per pizza. What should LP's price be in order to maximize its profit?



(c) On Wednesday, EP realizes what LP is doing: LP is maximizing its profit by setting its price after EP's price is set. EP continues to set its price first and sets a price that is a multiple of 20 cents. LP's price is still a multiple of 10 cents and the number of customers at each restaurant still follows the rule above. Determine the two prices that EP could charge in order to maximize its profit. State LP's profit in each case.



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Thank you for writing the 2016 Hypatia Contest! Each year, more than 220 000 students from more than 60 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2016.

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- Find your school's contest results