The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Grade 8 solutions follow the Grade 7 solutions

2013 Gauss Contests

(Grades 7 and 8)

Wednesday, May 15, 2013
(in North America and South America)

Thursday, May 16, 2013
(outside of North America and South America)

Solutions

Centre for Education in Mathematics and Computing Faculty and Staff

Ed Anderson
Jeff Anderson
Terry Bae
Steve Brown
Ersal Cahit
Serge D'Alessio
Frank DeMaio
Jennifer Doucet
Fiona Dunbar
Mike Eden
Barry Ferguson
Barb Forrest
Judy Fox
Steve Furino
John Galbraith
Sandy Graham
Angie Hildebrand
Judith Koeller
Joanne Kursikowski
Bev Marshman
Dean Murray
Jen Nissen
J.P. Pretti
Linda Schmidt
Kim Schnarr
Jim Schurter
Carolyn Sedore
Ian VanderBurgh
Troy Vasiga

Gauss Contest Committee

Mark Bredin (Chair), St. John's Ravenscourt School, Winnipeg, MB
Kevin Grady (Assoc. Chair), Cobden District P.S., Cobden, ON
Sarah Garrett, King George P.S., Guelph, ON
John Grant McLoughlin, University of New Brunswick, Fredericton, NB
JoAnne Halpern, Thornhill, ON
David Matthews, University of Waterloo, Waterloo, ON
David Switzer, Sixteenth Ave. P.S., Richmond Hill, ON
Rachael Verbruggen, University of Waterloo, Waterloo, ON
Laurissa Werhun, Parkdale C.I., Toronto, ON
Chris Wu, Zion Heights J.H.S., Toronto, ON
Lori Yee, William Dunbar P.S., Pickering, ON

Grade 7

1. Evaluating, $(5 \times 3) - 2 = 15 - 2 = 13$.

 Answer: (E)

2. *Solution 1*
 A number is a multiple of 9 if it is the result of multiplying 9 by an integer.
 Of the answers given, only 45 results from multiplying 9 by an integer, since $45 = 9 \times 5$.

 Solution 2
 A number is a multiple of 9 if the result after dividing it by 9 is an integer.
 Of the answers given, only 45 results in an integer after dividing by 9, since $45 \div 9 = 5$.

 Answer: (D)

3. Thirty-six hundredths equals $\frac{36}{100}$ or 0.36.

 Answer: (A)

4. By grouping terms using brackets as shown, $(1 + 1 - 2) + (3 + 5 - 8) + (13 + 21 - 34)$, we can see that the result inside each set of brackets is 0.
 Thus the value of $1 + 1 - 2 + 3 + 5 - 8 + 13 + 21 - 34$ is 0.

 Answer: (D)

5. Since PQ is a straight line segment, the three angles given sum to 180°.
 That is, $90^\circ + x^\circ + 20^\circ = 180^\circ$ or $x^\circ = 180^\circ - 90^\circ - 20^\circ$ or $x = 70$.

 Answer: (B)

6. Nick has 6 nickels and each nickel is worth 5¢. So Nick has 6×5¢ or 30¢ in nickels.
 Nick has 2 dimes and each dime is worth 10¢. So Nick has 2×10¢ or 20¢ in dimes.
 Nick has 1 quarter and each quarter is worth 25¢. So Nick has 1×25¢ or 25¢ in quarters.
 In total, Nick has 30¢ + 20¢ + 25¢ or 75¢.

 Answer: (B)

7. *Solution 1*
 To determine the smallest number in the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{6}, \frac{7}{12}\right\}$, we express each number with a common denominator of 12. The set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{6}, \frac{7}{12}\right\}$ is equivalent to the set $\left\{\frac{1\times6}{2\times6}, \frac{2\times4}{3\times4}, \frac{1\times3}{4\times3}, \frac{5\times2}{6\times2}, \frac{7}{12}\right\}$ or to the set $\left\{\frac{6}{12}, \frac{8}{12}, \frac{3}{12}, \frac{10}{12}, \frac{7}{12}\right\}$.
 The smallest number in this set is $\frac{3}{12}$, so $\frac{1}{4}$ is the smallest number in the original set.

 Solution 2
 With the exception of $\frac{1}{4}$, each number in the set is greater than or equal to $\frac{1}{2}$.
 We can see this by recognizing that the numerator of each fraction is greater than or equal to one half of its denominator.
 Thus, $\frac{1}{4}$ is the only number in the list that is less than $\frac{1}{2}$ and so it must be the smallest number in the set.

 Answer: (C)

8. Since Ahmed stops to talk with Kee one quarter of the way to the store, then the remaining distance to the store is $1 - \frac{1}{4} = \frac{3}{4}$ of the total distance.
 Since $\frac{3}{4} = 3 \times \frac{1}{4}$, then the distance that Ahmed travelled from Kee to the store is 3 times the distance that Ahmed travelled from the start to reach Kee.
 That is, 12 km is 3 times the distance between the start and Kee. So the distance between the start and Kee is $\frac{12}{3} = 4$ km. Therefore, the total distance travelled by Ahmed is $4 + 12$ or 16 km.

 Answer: (B)

9. When $n = 4$, we are looking for an expression that produces a value of 7.
The results of substituting $n = 4$ into each expression and evaluating are shown below.

Expression	Value
(A) $3n - 2$	$3(4) - 2 = 12 - 2 = 10$
(B) $2(n - 1)$	$2(4 - 1) = 2(3) = 6$
(C) $n + 4$	$4 + 4 = 8$
(D) $2n$	$2(4) = 8$
(E) $2n - 1$	$2(4) - 1 = 8 - 1 = 7$

Since $2n - 1$ is the only expression which gives a value of 7 when $n = 4$, it is the only possible answer. We check that the expression $2n - 1$ does give the remaining values, $1, 3, 5, 9$, when $n = 1, 2, 3, 5$, respectively.
(Alternately, we may have began by substituting $n = 1$ and noticing that this eliminates answers (B), (C) and (D). Substituting $n = 2$ eliminates (A). Substituting $n = 3$, $n = 4$ and $n = 5$ confirms the answer (E).)

Answer: (E)

10. To make the difference $UVW - XYZ$ as large as possible, we make UVW as large as possible and XYZ as small as possible.
The hundreds digit of a number contributes more to its value than its tens digit, and its tens digit contributes more to its value than its units digit.
Thus, we construct the largest possible number UVW by choosing 9 (the largest digit) to be its hundreds digit, U, and by choosing 8 (the second largest digit) to be its tens digit, V, and by choosing 7 (the third largest digit) to be the units digit, W.
Similarly, we construct the smallest possible number XYZ by choosing 1 (the smallest allowable digit) to be its hundreds digit, X, and 2 (the second smallest allowable digit) to be its tens digit, Y, and by choosing 3 (the third smallest allowable digit) to be its units digit, Z.
The largest possible difference is $UVW - XYZ$ or $987 - 123$ or 864.

Answer: (B)

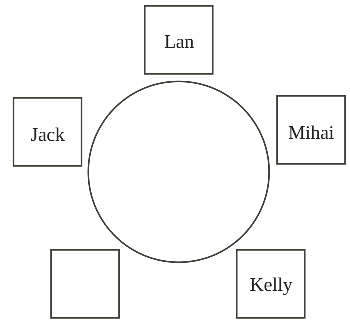
11. Each face of a cube is a square. The dimensions of each face of the cube are 1 cm by 1 cm. Thus, the area of each face of the cube is $1 \times 1 = 1\,\text{cm}^2$.
Since a cube has 6 identical faces, the surface area of the cube is $6 \times 1 = 6\,\text{cm}^2$.

Answer: (E)

12. The greatest common factor of two numbers is the largest positive integer which divides into both numbers with no remainder.
For answer (B), 50 divided by 20 leaves a remainder, so we may eliminate (B) as a possible answer.
Similarly for answer (D), 25 divided by 20 leaves a remainder, so we may eliminate (D) as a possible answer.
For answer (A), since 200 divides 200 and 200 divides 2000, the greatest common factor of 200 and 2000 cannot be 20.
For answer (E), since 40 divides 40 and 40 divides 80, the greatest common factor of 40 and 80 cannot be 20.
The largest positive integer which divides both 20 and 40 is 20, and so (C) is the correct answer.

Answer: (C)

13. Since Lan and Mihai are seated beside each other, while Jack and Kelly are not, the only possible location for the remaining chair (Nate's chair) is between Jack and Kelly, as shown. Therefore, the 2 people who are seated on either side of Nate are Jack and Kelly.



ANSWER: (B)

14. Substituting $x = 4$ into $3x + 2y$ we get, $3(4) + 2y = 12 + 2y$.
Since this expression $12 + 2y$ is equal to 30, then $2y$ must equal $30 - 12$ or 18.
If $2y = 18$, then y is $18 \div 2$ or 9.

ANSWER: (E)

15. Each time Daniel reaches into the jar, he removes half of the coins that are in the jar.
Since he removes half of the coins, then the other half of the coins remain in the jar.
We summarize Daniel's progress in the table below.

Number of Times Coins are Removed	0	1	2	3	4	5	6
Number of Coins Remaining in the Jar	64	32	16	8	4	2	1

For exactly 1 coin to remain in the jar, Daniel must reach in and remove coins from the jar 6 times.

ANSWER: (C)

16. *Solution 1*
Consider the set of five consecutive even numbers $8, 10, 12, 14, 16$.
The mean of these five numbers is $\frac{8+10+12+14+16}{5} = \frac{60}{5}$ or 12.
If the five consecutive even numbers were smaller, their mean would be less than 12.
If the five consecutive even numbers were larger, their mean would be greater than 12.
Therefore, this is the set of five consecutive even numbers that we seek.
The mean of the smallest and largest of these five numbers is $\frac{8+16}{2} = \frac{24}{2}$ or 12.

Solution 2
The mean of five consecutive even numbers is the middle (third largest) number.
To see this, consider that the smallest of the five numbers is 4 less than the middle number, while the largest of the five numbers is 4 more than the middle number.
Thus, the mean of the smallest and largest numbers is the middle number.
Similarly, the second smallest of the five numbers is 2 less than the middle number, while the fourth largest of the five numbers is 2 more than the middle number.
Thus, the mean of these two numbers is also the middle number.
Since the mean of the five numbers is 12, then the middle number is 12.
Therefore, the mean of the largest and smallest of the five numbers is also 12.

ANSWER: (A)

17. For every 3 chocolates that Claire buys at the regular price, she buys a fourth for 25 cents.
Consider dividing the 12 chocolates that Claire buys into 3 groups of 4 chocolates.
In each group of 4, Claire buys 3 chocolates at the regular price and the fourth chocolate is

purchased for 25 cents.
That is, of the 12 chocolates that Claire buys, 3 are bought at 25 cents each while the remaining 9 are purchased at the regular price.
The total cost to purchase 3 chocolates at 25 cents each is $3 \times 25 = 75$ cents.
Since Claire spent \$6.15 and the total cost of the 25 cent chocolates was 75 cents, then the cost of the regular price chocolates was $\$6.15 - \$0.75 = \$5.40$.
Since 9 chocolates were purchased at the regular price for a total of \$5.40, then the regular price of one chocolate is $\frac{\$5.40}{9} = \$0.60$ or 60 cents.

Answer: (C)

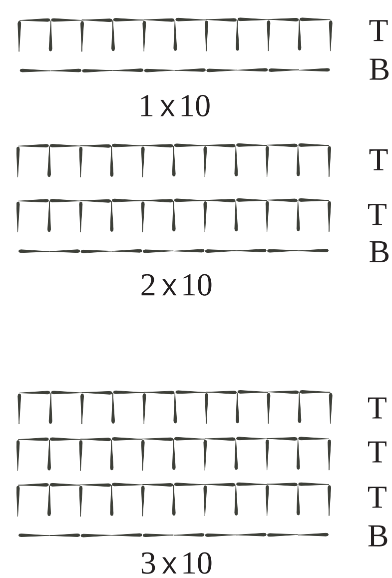
18. The total area of the shaded regions is the difference between the area of square $JKLM$ and the area of the portion of rectangle $PQRS$ that overlaps $JKLM$.
Since $JK = 8$, then the area of square $JKLM$ is 8×8 or 64.
Since JK is parallel to PQ, then the portion of $PQRS$ that overlaps $JKLM$ is a rectangle, and has length equal to JK or 8, and width equal to PS or 2.
So the area of the portion of $PQRS$ that overlaps $JKLM$ is $8 \times 2 = 16$.
Therefore the total area of the shaded regions is $64 - 16 = 48$.

Answer: (D)

19. Using the special six-sided die, the probability of rolling a number that is a multiple of three is $\frac{1}{2}$.
Since $\frac{1}{2}$ of 6 is 3, then exactly 3 numbers on the die must be multiples of 3.
Since the probability of rolling an even number is $\frac{1}{3}$ and $\frac{1}{3}$ of 6 is 2, then exactly 2 numbers on the die must be even.
The die in (A) has only 2 numbers that are multiples of 3 (3 and 6), and thus may be eliminated.
The die in (C) has 4 numbers that are even $(2, 4, 6, 6)$, and thus may be eliminated.
The die in (D) has 3 numbers that are even $(2, 4, 6)$, and thus may be eliminated.
The die in (E) has 4 numbers that are multiples of 3 $(3, 3, 3, 6)$, and thus may be eliminated.
The die in (B) has exactly 3 numbers that are multiples of 3 $(3, 3, 6)$, and exactly 2 even numbers (2 and 6), and is therefore the correct answer.

Answer: (B)

20. In the diagram shown, the 31 identical toothpicks used in the 1×10 grid are separated into 2 sections.
The top section, T, is made from 11 vertical toothpicks and 10 horizontal toothpicks, or 21 toothpicks in total.
The bottom section, B, is made of 10 horizontal toothpicks.
The 2×10 and 3×10 grids are similarly separated into top and bottom sections, as shown.
We observe that the 1×10 grid consists of 1 top section and 1 bottom.
The 2×10 grid consists of 2 top sections and 1 bottom.
The 3×10 grid consists of 3 top sections and 1 bottom.
Continuing in this way, a grid of size $n \times 10$ will consist of n top sections and 1 bottom section, for any positive integer n.
So then a grid of size 43×10 consists of 43 top sections and 1 bottom section.
Each top section is made from 21 toothpicks and each bottom section is made from 10 toothpicks.
Thus, the total number of toothpicks in a 43×10 grid is $(43 \times 21) + (1 \times 10)$ or $903 + 10$ or 913.



Answer: (A)

21. The sum of the units column is $P + P + P = 3P$.
Since P is a single digit, and $3P$ ends in a 7, then the only possibility is $P = 9$.
This gives:

$$\begin{array}{cccc} & 7 & 7 & 9 \\ & 6 & Q & 9 \\ + & Q & Q & 9 \\ \hline 1 & 9 & 9 & 7 \end{array}$$

Then $3P = 3 \times 9 = 27$, and thus 2 is carried to the tens column.
The sum of the tens column becomes $2 + 7 + Q + Q$ or $9 + 2Q$.
Since $9 + 2Q$ ends in a 9 (since $P = 9$), then $2Q$ ends in $9 - 9 = 0$.
Since Q is a single digit, there are two possibilities for Q such that $2Q$ ends in 0.
These are $Q = 0$ and $Q = 5$.
If $Q = 0$, then the sum of the tens column is 9 with no carry to the hundreds column.
In this case, the sum of the hundreds column is $7 + 6 + Q$ or 13 (since $Q = 0$); the units digit of this sum does not match the 9 in the total.
Thus, we conclude that Q cannot equal 0 and thus must equal 5.
Verifying that $Q = 5$, we check the sum of the tens column again.
Since $2 + 7 + 5 + 5 = 19$, then 1 is carried to the hundreds column.
The sum of the hundreds column is $1 + 7 + 6 + 5 = 19$, as required.
Thus, $P + Q = 9 + 5 = 14$ and the completed addition is shown below.

$$\begin{array}{cccc} & 7 & 7 & 9 \\ & 6 & 5 & 9 \\ + & 5 & 5 & 9 \\ \hline 1 & 9 & 9 & 7 \end{array}$$

Answer: (C)

22. We use labels, m and n, in the fourth row of the grid, as shown.
Then, $10, m, 36, n$ are four terms of an arithmetic sequence.
Since 10 and 36 are two terms apart in this sequence, and their difference is $36 - 10 = 26$, the constant added to one term to obtain the next term in the fourth row is $\frac{26}{2}$ or 13.
That is, $m = 10 + 13 = 23$, and $n = 36 + 13 = 49$.
(We confirm that the terms $10, 23, 36, 49$ do form an arithmetic sequence.)
In the fourth column, 25 and n (which equals 49) are two terms apart in this sequence, and their difference is $49 - 25 = 24$.
Thus, the constant added to one term to obtain the next term in the fourth column is $\frac{24}{2}$ or 12.
That is, $x = 25 + 12 = 37$ (or $x = 49 - 12 = 37$).
The completed grid is as shown.

1			
4			25
7			x
10	m	36	n

1	5	9	13
4	11	18	25
7	17	27	37
10	23	36	49

Answer: (A)

23. Since $\triangle PQR$ is isosceles with $PQ = QR$ and $\angle PQR = 90^\circ$, then $\angle QPR = \angle QRS = 45^\circ$.
Also in $\triangle PQR$, altitude QS bisects PR ($PS = SR$) forming two identical triangles, SQP and SQR.

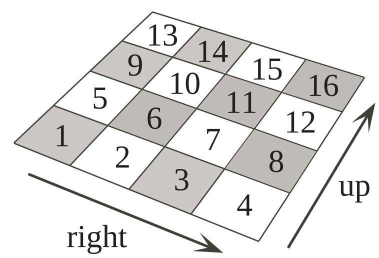
Since these two triangles are identical, each has $\frac{1}{2}$ of the area of $\triangle PQR$.

In $\triangle SQR$, $\angle QSR = 90°$, $\angle QRS = 45°$, and so $\angle SQR = 45°$.
Thus, $\triangle SQR$ is also isosceles with $SQ = SR$.
Then similarly, altitude ST bisects QR ($QT = TR$) forming two identical triangles, SQT and SRT.
Since these two triangles are identical, each has $\frac{1}{2}$ of the area of $\triangle SQR$ or $\frac{1}{4}$ of the area of $\triangle PQR$.

Continuing in this way, altitude TU divides $\triangle STR$ into two identical triangles, STU and RTU.
Each of these two triangles has $\frac{1}{2}$ of $\frac{1}{4}$ or $\frac{1}{8}$ of the area of $\triangle PQR$.
Continuing, altitude UV divides $\triangle RTU$ into two identical triangles, RUV and TUV.
Each of these two triangles has $\frac{1}{2}$ of $\frac{1}{8}$ or $\frac{1}{16}$ of the area of $\triangle PQR$.
Finally, altitude VW divides $\triangle RUV$ into two identical triangles, UVW and RVW.
Each of these two triangles has $\frac{1}{2}$ of $\frac{1}{16}$ or $\frac{1}{32}$ of the area of $\triangle PQR$.
Since the area of $\triangle STU$ is $\frac{1}{8}$ of the area of $\triangle PQR$, and the area of $\triangle UVW$ is $\frac{1}{32}$ of the area of $\triangle PQR$, then the total fraction of $\triangle PQR$ that is shaded is $\frac{1}{8} + \frac{1}{32} = \frac{4+1}{32}$ or $\frac{5}{32}$.

Answer: (D)

24. We begin by numbering the checkerboard squares from 1 to 16, as shown, so that we may refer to each of them specifically.
We denote a move "up" by the letter U and a move "right" by R.
We will begin by determining which of the 16 squares will not be touched by the face with the circle and then proceed to show that the remaining squares will be touched by the face with the circle.
Since the cube begins on square 1 and the circle is facing out, square 1 will not be touched by the face with the circle.



Similarly, each of the squares 2, 3, 4 can only be reached by moving the cube right (the sequence of moves to reach each of these three squares is R, RR and RRR, respectively), and in each case the circle remains facing out.
Squares 2, 3 and 4 will not be touched by the face with the circle.
Squares 5 and 9 can only be reached by moving the cube up (the sequence of moves to reach each of these two squares is U and UU, respectively).
In either case, the face with the circle will not touch squares 5 and 9.
Square 6 can be reached with two different sequences of moves, RU or UR.
In both cases, the face with the circle will not touch square 6.
Square 10 can be reached with three different sequences of moves, UUR, URU or RUU.
In all three cases, the face with the circle will not touch square 10.
In turns out that these eight squares $(1, 2, 3, 4, 5, 6, 9, 10)$ are the only squares that will not be touched by the face with the circle on any path.
The table below lists sequences of moves that demonstrate how each of the remaining eight squares will be touched by the face with the circle.
The second column lists the sequence of moves, while the third column lists the position of the face with the circle as the cube progresses through the sequence of moves.
We have used the letters F for front, B for back, T for top, O for bottom, L for left, and R for right to indicate the location of the face containing the circle.

Square	Sequence of Moves	Position of the Circle
7	URR	TRO
8	$RURR$	$FTRO$
11	$URUR$	$TRRO$
12	$RURUR$	$FTRRO$
13	UUU	TBO
14	$RUUU$	$FTBO$
15	$RRUUU$	$FFTBO$
16	$RRRUUU$	$FFFTBO$

Therefore, the number of different squares that will not be contacted by the face with the circle on any path is 8.

Answer: (C)

25. We denote the number of tickets of each of the five colours by the first letter of the colour.
We are given that $b : g : r = 1 : 2 : 4$ and that $g : y : o = 1 : 3 : 6$.
Through multiplication by 2, the ratio $1 : 3 : 6$ is equivalent to the ratio $2 : 6 : 12$.
Thus, $g : y : o = 2 : 6 : 12$.
We chose to scale this ratio by a factor of 2 so that the only colour common to the two given ratios, green, now has the same number in both of these ratios.
That is, $b : \mathbf{g} : r = 1 : \mathbf{2} : 4$ and $\mathbf{g} : y : o = \mathbf{2} : 6 : 12$ and since the term g is 2 in each ratio, then we can combine these to form a single ratio, $b : g : r : y : o = 1 : 2 : 4 : 6 : 12$.
This ratios tells us that for every blue ticket, there are 2 green, 4 red, 6 yellow, and 12 orange tickets.
Thus, if there was only 1 blue ticket, then there would be $1 + 2 + 4 + 6 + 12 = 25$ tickets in total.
However, we are given that the box contains 400 tickets in total.
Therefore, the number of blue tickets in the box is $\frac{400}{25} = 16$.
Through multiplication by 16, the ratio $b : g : r : y : o = 1 : 2 : 4 : 6 : 12$ becomes $b : g : r : y : o = 16 : 32 : 64 : 96 : 192$.
(Note that there are $16 + 32 + 64 + 96 + 192 = 400$ tickets in total.)
Next, we must determine the smallest number of tickets that must be drawn to ensure that at least 50 tickets of one colour have been selected.
It is important to consider that up to 49 tickets of any one colour could be selected without being able to *ensure* that 50 tickets of one colour have been selected.
That is, it is possible that the first 195 tickets selected could include exactly 49 orange, 49 yellow, 49 red, all 32 green, and all 16 blue tickets $(49 + 49 + 49 + 32 + 16 = 195)$.
Since all green and blue tickets would have been drawn from the box, the next ticket selected would have to be the 50^{th} orange, yellow or red ticket.
Thus, the smallest number of tickets that must be drawn to ensure that at least 50 tickets of one colour have been selected is 196.

Answer: (D)

Grade 8

1. Evaluating, $10^2 + 10 + 1 = 10 \times 10 + 10 + 1 = 100 + 10 + 1 = 111$.

   ANSWER: (D)

2. Evaluating, $15 - 3 - 15 = 12 - 15 = -3$.

   ANSWER: (D)

3. *Solution 1*
   To determine the smallest number in the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{6}, \frac{7}{12}\right\}$, we express each number with a common denominator of 12. The set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{6}, \frac{7}{12}\right\}$ is equivalent to the set $\left\{\frac{1\times6}{2\times6}, \frac{2\times4}{3\times4}, \frac{1\times3}{4\times3}, \frac{5\times2}{6\times2}, \frac{7}{12}\right\}$ or to the set $\left\{\frac{6}{12}, \frac{8}{12}, \frac{3}{12}, \frac{10}{12}, \frac{7}{12}\right\}$.
   The smallest number in this set is $\frac{3}{12}$, so $\frac{1}{4}$ is the smallest number in the original set.

   Solution 2
   With the exception of $\frac{1}{4}$, each number in the set is greater than or equal to $\frac{1}{2}$.
   We can see this by recognizing that the numerator of each fraction is greater than or equal to one half of its denominator.
   Thus, $\frac{1}{4}$ is the only number in the list that is less than $\frac{1}{2}$ and so it must be the smallest number in the set.

   ANSWER: (C)

4. Since Ahmed stops to talk with Kee one quarter of the way to the store, then the remaining distance to the store is $1 - \frac{1}{4} = \frac{3}{4}$ of the total distance.
   Since $\frac{3}{4} = 3 \times \frac{1}{4}$, then the distance Ahmed travelled from Kee to the store, is 3 times the distance Ahmed travelled from the start to reach Kee.
   That is, 12 km is 3 times the distance between the start and Kee.
   So the distance between the start and Kee is $\frac{12}{3} = 4$ km.
   Therefore, the total distance travelled by Ahmed altogether is $4 + 12$ or 16 km.

   ANSWER: (B)

5. Since Jarek multiplies a number by 3 and gets an answer of 90, then the number must be $\frac{90}{3} = 30$. (We may check that $30 \times 3 = 90$.)
   If Jarek instead divides the number 30 by 3, then the answer he gets is $\frac{30}{3} = 10$.

   ANSWER: (B)

6. *Solution 1*
   We first evaluate the product on the left side of the equation.
   $10 \times 20 \times 30 \times 40 \times 50 = 200 \times 30 \times 40 \times 50 = 6000 \times 40 \times 50 = 240\,000 \times 50 = 12\,000\,000$.
   Similarly, we evaluate the product on the right side of the equation.
   $100 \times 2 \times 300 \times 4 \times \square = 200 \times 300 \times 4 \times \square = 60\,000 \times 4 \times \square = 240\,000 \times \square$.
   The left side of the equation is equal to the right side of the equation, so then
   $12\,000\,000 = 240\,000 \times \square$.
   Therefore, the number that goes in the box is $12\,000\,000 \div 240\,000 = 50$.

   Solution 2
   The product of the first two numbers on the left side of the equation is equal to the product of the first two numbers on the right side of the equation.
   That is, $10 \times 20 = 200 = 100 \times 2$.
   Similarly, the product of the next two numbers (the third and fourth numbers) on the left

side of the equation is equal to the product of the next two numbers on the right side of the equation.
That is, $30 \times 40 = 1200 = 300 \times 4$.
Since the products of the first four numbers on each side of the equation are equal, then the final (fifth) number on each side of the equation must be equal.
Therefore, the number that goes in the box is equal to the fifth number on the left side of the equation or 50.

Answer: (C)

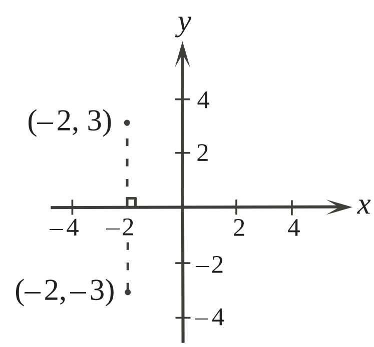
7. There are 26 letters in the English alphabet.
There are 5 different letters, a, l, o, n, s, in Alonso's name.
If one letter is randomly drawn from the bag, then the probability that it is a letter in Alonso's name is $\frac{5}{26}$.

Answer: (C)

8. When Mathy Manuel's autograph dropped 30% in value, it lost $\$100 \times 0.30 = \$30$ of its value.
After this drop, the autograph was worth $\$100 - \$30 = \$70$.
If the autograph then increased by 40% in value, the increase would be $\$70 \times 0.40 = \$28$.
After this increase, the autograph would be worth $\$70 + \$28 = \$98$.

Answer: (A)

9. After reflecting the point $(-2, -3)$ in the x-axis, the x-coordinate of the image will be the same as the x-coordinate of the original point, $x = -2$.
The original point is a distance of 3 below the x-axis.
The image will be the same distance from the x-axis, but above the x-axis.
Thus, the image has y-coordinate 3.
The coordinates of the image point are $(-2, 3)$.



Answer: (E)

10. The value of four nickels is $4 \times 5¢ = 20¢$.
The value of six dimes is $6 \times 10¢ = 60¢$.
The value of two quarters is $2 \times 25¢ = 50¢$.
The ratio of the value of four nickels to six dimes to two quarters is $20 : 60 : 50 = 2 : 6 : 5$.

Answer: (B)

11. Substituting $x = 4$ into $3x + 2y$ we get $3(4) + 2y = 12 + 2y$.
Since this expression $12 + 2y$ is equal to 30, then $2y$ must equal $30 - 12$ or 18.
If $2y = 18$, then y is $18 \div 2$ or 9.

Answer: (E)

12. *Solution 1*
Using order of operations to evaluate, $(2^3)^2 - 4^3 = 8^2 - 4^3 = 64 - 64 = 0$.

Solution 2
We first express 4 as 2^2 and then use exponent rules to evaluate.

$$\begin{aligned}(2^3)^2 - 4^3 &= (2^3)^2 - (2^2)^3 \\ &= 2^{3\times 2} - 2^{2\times 3} \\ &= 2^6 - 2^6 \\ &= 0\end{aligned}$$

Answer: (A)

13. Two consecutive Summer Olympics are held 4 years apart.
Each successive Summer Olympics requires an additional 4 years before it is held.
We use this to summarize the minimum time required for the largest number of Summer Olympics to be held, as shown.

Number of Summer Olympics	Minimum Number of Years Apart
2	4
3	8
4	12
5	16
6	20

From the table above, we see that at least 20 years are required to host 6 Summer Olympics, while only 16 years are required to host 5.
(For example, the 5 olympics could be held in years $1, 5, 9, 13$, and 17.)
Therefore, the maximum number of Summer Olympics that can be held during an 18 year period is 5.

Answer: (C)

14. Let s be the side length of the cube.
The surface area of a cube is made up of 6 identical squares.
Since the surface area is 54 cm^2, then each of the 6 squares has area $(54 \div 6)\,\text{cm}^2 = 9\,\text{cm}^2$.
The area of a square with side length s is s^2, so $s^2 = 9$ or $s = \sqrt{9} = 3$ cm.
The volume of a cube is given by the product of its length, width and height, which are all equal to s, 3 cm.
Thus the volume of the cube with surface area 54 cm^2 is $3 \times 3 \times 3 = 27$ cm^3.

Answer: (D)

15. *Solution 1*
When 10 000 is divided by 13 with the help of a calculator, we get $10\,000 \div 13 = 769.230\ldots$.
Since $769 \times 13 = 9997$ and $10\,000 - 9997 = 3$, then we have a quotient of 769 and a remainder of 3.
That is, $10\,000 = 769 \times 13 + 3$.
This is called a *division statement*.
Similarly, we may determine the remainder when each of the five possible answers (*the dividend*) is divided by 13 (*the divisor*).
We summarize this work in the table below.

Answer	Division Statement	Remainder
(A)	$9\,997 = 769 \times 13 + 0$	0
(B)	$10\,003 = 769 \times 13 + 6$	6
(C)	$10\,013 = 770 \times 13 + 3$	3
(D)	$10\,010 = 769 \times 13 + 0$	0
(E)	$10\,016 = 770 \times 13 + 6$	6

Of the five possible answers, only 10 013 gives a remainder of 3 when divided by 13.

Solution 2
When 10 000 is divided by 13, the remainder is 3.
Thus, adding any multiple of 13 to 10 000 will give the same remainder, 3, upon division by 13.
Of the five choices given, the only answer that differs from 10 000 by a multiple of 13 is 10 013.
When 10 013 is divided by 13, the remainder is also 3.

ANSWER: (C)

16. Since it is equally likely that a child is a boy as it is that a child is a girl, then the probability that any child is a girl is $\frac{1}{2}$.
The probability of any child being born a girl is independent of the number or gender of any children already born into the family.
That is, the probability of the second child being a girl is also $\frac{1}{2}$, as is the probability of the third child being a girl.
Therefore, the probability that all 3 children are girls is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

ANSWER: (E)

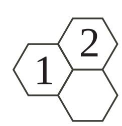
17. Since $PQRS$ is a rectangle, then $\angle PQR = 90^\circ$.
Since $\angle PQR = \angle PQS + \angle RQS$, then $90^\circ = (5x)^\circ + (4x)^\circ$ or $90 = 9x$ and so $x = 10$.
In $\triangle SRQ$, $\angle SRQ = 90^\circ$ and $\angle RQS = (4x)^\circ = 40^\circ$.
Therefore, $\angle QSR = y^\circ = 180^\circ - 90^\circ - 40^\circ = 50^\circ$.
So $y = 50$.

ANSWER: (D)

18. Sally's answer is $\frac{2}{3} \times 1\frac{1}{2} = \frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$.
Jane's answer is $\frac{2}{3} + 1\frac{1}{2} = \frac{2}{3} + \frac{3}{2} = \frac{4}{6} + \frac{9}{6} = \frac{13}{6} = 2\frac{1}{6}$.
The difference between Sally's answer and Jane's answer is $2\frac{1}{6} - 1$ or $1\frac{1}{6}$.

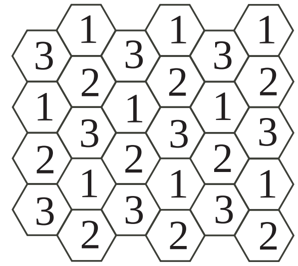
ANSWER: (B)

19. Since we seek the minimum number of colours that Serena can use to colour the hexagons, we first determine if it is possible for her to use only two colours (using only one colour is not possible). We will use the numbers $1, 2, 3$ to represent distinct (different) colours.
We begin by choosing any group of three hexagons in which each pair of hexagons share a side, as shown.
We colour two of the hexagons shown with colours 1 and 2 (since they share a side).



Each of these two coloured hexagons share a side with the third hexagon which therefore can not be coloured 1 or 2.
Thus, the minimum number of colours that Serena can use is at least three.

Next, we determine if the entire tiling can be coloured using only three colours.
One possible colouring of the tiles that uses only three colours is shown to the right.
While other colourings of the tiles are possible, Serena is able to use only three colours and ensure that no two hexagons that share a side are the same colour.
There are many nice patterns of the colours in this tiling.
Can you find a different colouring of the tiles that uses only three colours?



ANSWER: (E)

20. *Solution 1*
Suppose that the cost of one book, in dollars, is C.
Then Christina has $\frac{3}{4}C$ and Frieda has $\frac{1}{2}C$.
Combining their money, together Christina and Frieda have $\frac{3}{4}C + \frac{1}{2}C = \frac{3}{4}C + \frac{2}{4}C = \frac{5}{4}C$.
If the book was \$3 cheaper, then the cost to buy one book would be $C - 3$.
If the cost of one book was $C - 3$, then the cost to buy two at this price would be $2(C - 3)$ or $2C - 6$.
Combined, Christina and Frieda would have enough money to buy exactly two books at this reduced price.
Thus, $2C - 6 = \frac{5}{4}C$.
Solving,

$$\begin{aligned} 2C - 6 &= \tfrac{5}{4}C \\ 2C - \tfrac{5}{4}C &= 6 \\ \tfrac{8}{4}C - \tfrac{5}{4}C &= 6 \\ \tfrac{3}{4}C &= 6 \end{aligned}$$

Since $\frac{3}{4}$ of 8 is 6, then $C = 8$.
Therefore, the original price of the book is \$8.

Solution 2
We proceed by systematically trying the five multiple choice answers given.

Initial cost of the book	Combined money for Christina and Frieda	Reduced cost of the book	Number of books they may buy
\$4	$\frac{3}{4}$ of \$4 + $\frac{1}{2}$ of \$4 = \$3 + \$2 = \$5	\$1	$5 \div 1 = 5$
\$16	$\frac{3}{4}$ of \$16 + $\frac{1}{2}$ of \$16 = \$12 + \$8 = \$20	\$13	$20 \div 13 = 1.53\ldots$
\$12	$\frac{3}{4}$ of \$12 + $\frac{1}{2}$ of \$12 = \$9 + \$6 = \$15	\$9	$15 \div 9 = 1.66\ldots$
\$10	$\frac{3}{4}$ of \$10 + $\frac{1}{2}$ of \$10 = \$7.50 + \$5 = \$12.50	\$7	$12.50 \div 7 = 1.78\ldots$
\$8	$\frac{3}{4}$ of \$8 + $\frac{1}{2}$ of \$8 = \$6 + \$4 = \$10	\$5	$10 \div 5 = 2$

We see that if the original price of the book is \$8, Christina and Frieda are able to buy exactly two copies of the book at the reduced price.
Thus, the initial cost of the book is \$8.

ANSWER: (E)

21. We use labels m and n in the first column, as shown in the top grid.
Then, $5, m, n, 23$ are four terms of an arithmetic sequence.
Since 5 and 23 are three terms apart in this sequence, and their difference is $23 - 5 = 18$, the constant added to one term to obtain the next term in the first column is $\frac{18}{3}$ or 6.
That is, $m = 5 + 6 = 11$, and $n = 11 + 6 = 17$.
(We confirm that the terms $5, 11, 17, 23$ do form an arithmetic sequence.)

5			
m			1211
n		1013	
23	x		

We use labels p and q in the second row, as shown in the middle grid.
Then, $11, p, q, 1211$ are four terms of an arithmetic sequence.
Since 11 and 1211 are three terms apart in this sequence, and their difference is $1211 - 11 = 1200$, the constant added to one term to obtain the next term in the second row is $\frac{1200}{3}$ or 400.
That is, $p = 11 + 400 = 411$, and $q = 411 + 400 = 811$.
(We confirm that the terms $11, 411, 811, 1211$ do form an arithmetic sequence.)

5			
11	p	q	1211
17		1013	
23	x		

We use label r in the third row, as shown in the third grid.
Then, $17, r, 1013$ are three terms of an arithmetic sequence.
Since 17 and 1013 are two terms apart in this sequence, and their difference is $1013 - 17 = 996$, the constant added to one term to obtain the next term in the third row is $\frac{996}{2}$ or 498.
That is, $r = 17 + 498 = 515$.
(We confirm that the terms $17, 515, 1013$ do form an arithmetic sequence.)

5			
11	411	811	1211
17	r	1013	
23	x		

Finally, in the second column the terms $411, 515, x$ are three terms of an arithmetic sequence.
Since 411 and 515 are one term apart in this sequence, the constant added to one term to obtain the next term in the second column is $515 - 411 = 104$.
Then, $x = 515 + 104 = 619$.
The completed grid is as shown.

5	307	609	911
11	411	811	1211
17	515	1013	1511
23	619	1215	1811

ANSWER: (B)

22. In $\triangle FGH$, $FG = GH = x$ since they are both radii of the same circle.
By the Pythagorean Theorem, $FH^2 = FG^2 + GH^2 = x^2 + x^2$, or $FH^2 = 2x^2$, and so $(\sqrt{8})^2 = 2x^2$ or $2x^2 = 8$ and $x^2 = 4$, so then $x = 2$ (since $x > 0$).
FG, GH and arc FH form a sector of a circle with centre G and radius GH.
Since $\angle FGH = 90^\circ$, which is $\frac{1}{4}$ of 360°, then the area of this sector is one quarter of the area of the circle with centre G and radius $GH = FG = 2$.
The shaded area is equal to the area of sector FGH minus the area of $\triangle FGH$.
The area of sector FGH is $\frac{1}{4}\pi(2)^2$ or $\frac{1}{4}\pi(4)$ or π.
The area of $\triangle FGH$ is $\frac{FG \times GH}{2}$ or $\frac{2\times 2}{2}$, so 2.
Therefore, the area of the shaded region is $\pi - 2$.

ANSWER: (A)

23. *Solution 1*

In the first race, when Azarah crossed the finish line, Charlize was 20 m behind or Charlize had run 80 m.
Since Azarah and Charlize each travelled these respective distances in the same amount of time, then the ratio of their speeds is equal to the ratio of their distances travelled, or 100 : 80.
Similarly in the second race, when Charlize crossed the finish line, Greg was 10 m behind or Greg had run 90 m.
Since Charlize and Greg each travelled these respective distances in the same amount of time, then the ratio of their speeds is equal to the ratio of their distances travelled, or 100 : 90.
Let A, C and G represent Azarah's, Charlize's and Greg's speeds, respectively.
Then, $A : C = 100 : 80 = 25 : 20$ and $C : G = 100 : 90 = 20 : 18$.
Therefore, $A : C : G = 25 : 20 : 18$ and $A : G = 25 : 18 = 100 : 72$.
Over equal times, the ratio of their speeds is equal to the ratio of their distances travelled.
Therefore, when Azarah travels 100 m, Greg travels 72 m.
When Azarah crossed the finish line, Greg was $100 - 72 = 28$ m behind.

Solution 2

In the first race, when Azarah crossed the finish line, Charlize was 20 m behind or Charlize had run 80 m.
Since Azarah and Charlize each travelled these respective distances in the same amount of time, then the ratio of their speeds is equal to the ratio of their distances travelled, or 100 : 80.
That is, Charlize's speed is 80% of Azarah's speed.
Similarly, Greg's speed is 90% of Charlize's speed.
Therefore, Greg's speed is 90% of Charlize's speed which is 80% of Azarah's speed, or Greg's speed is 90% of 80% of Azarah's speed.
Since 90% of 80% is equivalent to $0.90 \times 0.80 = 0.72$ or 72%, then Greg's speed is 72% of Azarah's speed.
When Azarah ran 100 m (crossed the finish line), Greg ran 72% of 100 m or 72 m in the same amount of time.
When Azarah crossed the finish line, Greg was $100 - 72 = 28$ m behind.

ANSWER: (C)

24. The length of the longest side, z, is less than half of the perimeter 57.
Thus, $z < \frac{57}{2}$ or $z < 28\frac{1}{2}$. Since z is an integer then $z \leq 28$.
When $z = 28$, $x + y = 57 - 28 = 29$.
We list all possible values for x and y in the table below given that $z = 28$ and $x < y < z$.

y	27	26	25	24	23	22	21	20	19	18	17	16	15
x	2	3	4	5	6	7	8	9	10	11	12	13	14

Systematically, we continue decreasing the value of z and listing all possible values for x and y.
When $z = 27$, $x + y = 57 - 27 = 30$.

y	26	25	24	23	22	21	20	19	18	17	16
x	4	5	6	7	8	9	10	11	12	13	14

When $z = 26$, $x + y = 57 - 26 = 31$.

y	25	24	23	22	21	20	19	18	17	16
x	6	7	8	9	10	11	12	13	14	15

When $z = 25$, $x + y = 57 - 25 = 32$.

y	24	23	22	21	20	19	18	17
x	8	9	10	11	12	13	14	15

When $z = 24$, $x + y = 57 - 24 = 33$.

y	23	22	21	20	19	18	17
x	10	11	12	13	14	15	16

When $z = 23$, $x + y = 57 - 23 = 34$.

y	22	21	20	19	18
x	12	13	14	15	16

When $z = 22$, $x + y = 57 - 22 = 35$.

y	21	20	19	18
x	14	15	16	17

When $z = 21$, $x + y = 57 - 21 = 36$.

y	20	19
x	16	17

When $z = 20$, $x + y = 57 - 20 = 37$.

y	19
x	18

The next smallest value for z is 19 and in this case $x + y = 57 - 19 = 38$.
However, if $x + y = 38$ then at least one of x or y must be 19 or larger.
This is not possible since $z = 19$ and $x < y < z$.
Therefore, 20 is the smallest possible value for z and we have listed the side lengths of all possible triangles above.
Counting, we see that there are $13+11+10+8+7+5+4+2+1 = 61$ possible triangles that satisfy the given conditions.

ANSWER: (B)

25. *Solution 1*

Let b represent the number of boys initially registered in the class.
Let g represent the number of girls initially registered in the class.
When 11 boys transferred into the class, the number of boys in the class was $b + 11$.
When 13 girls transferred out of the class, the number of girls in the class was $g - 13$.
The ratio of boys to girls in the class at this point was $1 : 1$.
That is, the number of boys in the class was equal to the number of girls in the class, or $b + 11 = g - 13$ and so $g = b + 24$.
Since there were at least 66 students initially registered in the class, then $b + g \geq 66$.
Substituting $g = b + 24$, $b + g$ becomes $b + (b + 24) = 2b + 24$, and so $2b + 24 \geq 66$ or $2b \geq 42$, so $b \geq 21$.
At this point, we may use the conditions that $g = b + 24$ and $b \geq 21$ to determine which of the 5 answers given is not possible.
Each of the 5 answers represents a possible ratio of $b : g$ or $b : (b + 24)$ (since $g = b + 24$).
We check whether $b : (b + 24)$ can equal each of the given ratios while satisfying the condition that $b \geq 21$.
In (A), we have $b : (b + 24) = 4 : 7$ or $\frac{b}{b+24} = \frac{4}{7}$ or $7b = 4b + 96$ and then $3b = 96$, so $b = 32$.
In (B), we have $b : (b + 24) = 1 : 2$ or $\frac{b}{b+24} = \frac{1}{2}$ or $2b = b + 24$, so $b = 24$.
In (C), we have $b : (b + 24) = 9 : 13$ or $\frac{b}{b+24} = \frac{9}{13}$ or $13b = 9b + 216$ and then $4b = 216$, so $b = 54$.
In (D), we have $b : (b + 24) = 5 : 11$ or $\frac{b}{b+24} = \frac{5}{11}$ or $11b = 5b + 120$ and then $6b = 120$, so $b = 20$.

In (E), we have $b : (b+24) = 3 : 5$ or $\frac{b}{b+24} = \frac{3}{5}$ or $5b = 3b + 72$ and then $2b = 72$, so $b = 36$.
Thus, the only ratio that does not satisfy the condition that $b \geq 21$ is $5 : 11$ (since $b = 20$).

Solution 2

As in Solution 1, we establish the conditions that $g = b + 24$ and $b \geq 21$.
Again, the ratio $b : g$ or $\frac{b}{g}$ then becomes $\frac{b}{b+24}$ (since $g = b + 24$).
Since $b \geq 21$, then b can equal $21, 22, 23, 24, \ldots$, but the smallest possible value for b is 21.
When $b = 21$, $\frac{b}{b+24}$ becomes $\frac{21}{21+24} = \frac{21}{45} = 0.4\overline{6}$.
When $b = 22$, $\frac{b}{b+24}$ becomes $\frac{22}{22+24} = \frac{22}{46} \approx 0.478$.
When $b = 23$, $\frac{b}{b+24}$ becomes $\frac{23}{23+24} = \frac{23}{47} \approx 0.489$.
When $b = 24$, $\frac{b}{b+24}$ becomes $\frac{24}{24+24} = \frac{24}{48} = 0.5$.
As b continues to increase, the value of the ratio $\frac{b}{b+24}$ or $\frac{b}{g}$ continues to increase.
Can you verify this for yourself?
Thus, the smallest possible value of the ratio $\frac{b}{g}$ is $\frac{21}{45}$ or $0.4\overline{6}$.
Comparing this ratio to the 5 answers given, we determine that $\frac{5}{11} = 0.\overline{45} < 0.4\overline{6} = \frac{21}{45}$, which is not possible.
(You may also confirm that each of the other 4 answers given are all greater than $\frac{21}{45}$ and are obtainable, as in Solution 1.)
Therefore, the only ratio of boys to girls which is not possible is (D).

Answer: (D)